Airbourne Equations

Fundamentals of Traffic Crash Reconstruction _Daily-Shigemura-Daily_: Chapter 15

Base units are lb, slugs, ft, ft/s. g = 32.2 ft/sec^2. S is speed in mph. 1 mph = 1.466 fps. v_0 = 1.466 mph

Motion of center of mass after rear wheels leave ground.
d is the distance where first contacted ground,
h is the vertical distance in ft of the COM displaced (+ lower, - higher) from ejection
θ is takeoff angle. Not usually known. **Approx as 45 deg**. 50 deg pedestrian goes over hood.
A launch angle of 45 deg gives the furthest range and the slowest launch velocity
m is the slope Δh/Δd, where h has the Sign/direction of g, **+** below ground, **-** above ground.
m = tanθ

Find Initial Velocity, v_0

Given d,θ, h pg 494)

Base units

$$v_0(d,\theta,h) := \frac{4.01\cdot d}{\cos(\theta\cdot deg)\cdot\sqrt{h + d\cdot\tan(\theta\cdot deg)}}$$

divide both sides by 1.466 to get S, speed in mph

$$S_0(d,\theta,h) := \frac{2.73\cdot d}{\cos(\theta\cdot deg)\cdot\sqrt{h + d\cdot\tan(\theta\cdot deg)}}$$

Find Final Velocity, vf, Magnitude and Direction/Angle

Given v_0, h (DSD pg 497)

$$v_f(v_0,h) := \sqrt{{v_0}^2 + 2g\cdot h} \qquad \theta(v_x,v_y) := \operatorname{acos}\left(\frac{v_x}{v_y}\right)$$

Find Maximum Height and Range at Maximum Height

Given v_0 and angle (DSD pg 498)

Maximum height occurs when vertical velocity equals 0

$$h_{max}(v_0,\theta) := \frac{-\left({v_0}^2\cdot\sin(\theta\cdot deg)^2\right)}{2g} \qquad d(v_0,\theta) := {v_0}^2\sin(\theta\cdot deg)\cdot\cos(\theta\cdot deg)$$

Find Maximum Range if Takeoff V is Known

Given v_0 h, and angle (DSD pg 499) Sign is - for landing heights above launch point

Sign := 1

$$d(v_0,h,\theta) := \frac{{v_0}^2\sin(2\theta\cdot deg) + Sign\sqrt{{v_0}^2\sin(2\theta\cdot deg)^2 + 8g\cdot h\cdot\cos(\theta\cdot deg)^2}}{2g}$$

$$d(S_0,h,\theta) := \frac{{S_0}^2\sin(2\theta\cdot deg)}{30} + Sign\cdot 0.02277S_0\sqrt{2.15{S_0}^2\sin(2\theta\cdot deg)^2 + 257.6\cdot h\cdot\cos(\theta\cdot deg)^2}$$

Special Cases: **If Takeoff Angle = 0**

$$d(v_0,\theta) := Sign\cdot v_0\cdot\sqrt{\frac{2h}{g}}$$

If h = 0

$$d(v_0,\theta) := \frac{{v_0}^2\sin(2\theta\cdot deg)}{g}$$

Find Angle that will Give Minimum Launch Velocity, S

Given d and h

$$\theta_{Vmin}(d,h) := \frac{1}{2}\operatorname{atan}\left(\frac{d}{h}\right)\frac{1}{deg} \qquad S_0(d,\theta,h) := \frac{2.73\cdot d}{\cos(\theta\cdot deg)\cdot\sqrt{h + d\cdot\tan(\theta\cdot deg)}}$$

For parabolic path: h = ad^2 + bd, where h is y and d is x, Find the Launch Angle, given 2 pts.

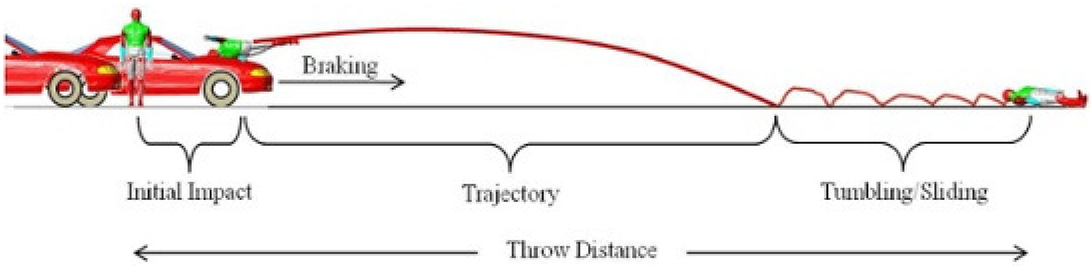
$$a(d_1,d_2,h_1,h_2) := \frac{d_2\cdot h_1 - d_1\cdot h_2}{{d_1}^2\cdot d_2 - {d_2}^2\cdot d_1} \qquad b(d_1,d_2,h_1,h_2) := \frac{{d_1}^2\cdot h_2 - {d_2}^2\cdot h_1}{{d_1}^2\cdot d_2 - {d_2}^2\cdot d_1}$$

$$m = 2a\cdot d + b$$

$$\theta(m) := -\operatorname{atan}(-m)\cdot deg^{-1}$$

$$\theta(0.4056) = 22.077$$

Estimating Vault Distance and Speed after a motorcyclist ejection - Taro - A Rich



Assume vault speed = slide to stop speed

$$v_{final} := \sqrt{2 \cdot 80 \cdot ft \cdot 0.5g} = 34.591 \cdot mph$$

Final Velocity: Initial Velocity, acceleration, and distance

$v_f = v_0 + a \cdot t$ $\quad$ $v_f(v_o, a, d) := \sqrt{v_o^2 + 2a \cdot d}$